

# Time Value of Money and Capital Budgeting

by

Professor M.Rammohan Rao

Former Dean

Professor Emeritus

Executive Director, Centre for

Analytical Finance

Indian School of Business, Hyderabad

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# Overview

## ➤ Time Value of Money

- Compounding

- Annuity

  - Perpetuity

  - Growth

## ➤ Capital Budgeting

- Selection Criteria

  - Financial

    - Payback

- Discounted Payback
- Net Present Value
- Internal Rate of Return
- Benefit Cost Ratio
- Inflation
- Cost Concepts
- Multi Criteria Models
  - Unweighted 0 -1 factor model
  - Unweighted factor scoring
  - Weighted factor scoring

# Time Value of Money

## ➤ One period investment

- Project ends after one year

Investment of Rs.1000

Amount received after one year:  
Rs.1100

Return on investment

$$= (1100 - 1000) / 1000$$

$$= 0.1 \text{ or } 10\%$$

- The amount to be received after one year may not be known with certainty

If Rs.1100 is the expected amount after one year, then the expected return = 10%

Rs.1100 to be received one year hence. What is it worth today?

Discount rate depends upon whether the amount to be received is known with certainty or not.

Suppose the discount rate is 8%

Then Present Value, PV, of  
future cash flow of 1100 is  
 $(1100) / (1.08) = 1018.52$

Invest in the project since PV of  
Future cash flow > Cost of  
investment

# Compounding

❖ Suppose Rs.100 invested earns 8% compounded annually for two years

Then at the end of one year, the investment will be worth

$$100 (1.08) = 108$$



❖ At the end of two years, the investment will be worth

$$108 (1.08) = 100 (1.08) (1.08) \\ = 116.64$$

$$\text{Or} \quad = 100 (1.08)^2 \\ = 100 (1.1664) \\ = 116.64$$

## ❖ Semiannual Compounding

Suppose Rs.100 invested earns 4% compounded semi-annually.

At the end of one year, the investment will be worth

$$\begin{aligned} 100 (1.04) (1.04) &= 100 (1.0816) \\ &= 108.16 \end{aligned}$$

Note: The annual return is 8.16% which is greater than 8%

## ❖ Quarterly Compounding

Suppose Rs.100 invested earns 2% compounded quarterly.

At the end of one year, the investment will be worth

$$100 (1.02)^4 = 100 (1.0824) = 108.24$$

Note: The annual return is 8.24% which is greater than 8.16%

## ❖ Continuous Compounding

Suppose  $r$  is the stated annual rate compounded continuously.

Then Rs.100 invested for  $T$  years with continuous compounding will, at the end of  $T$  years, be worth  $100 e^{rT}$  where  $e = 2.718$

Suppose  $r = 8\%$  and  $T = 1$  year, then  $100 e^{0.08(1)} = 108.33$  or a return of  $8.33\%$

If  $T = 2$ ,  $100 e^{0.08(2)} = 117.35$

## ❖ Power of compounding

Suppose Rs.100 invested earns 10% compounded annually for say 50 years.

At the end of 50 years, the investment will be worth

$$100 (1.1)^{50} = 11,739.09$$

## ❖ Annuity into Perpetuity

An amount  $A$  is received annually one year from now into perpetuity.

$$\begin{aligned}\text{Present Value, PV} &= [A / (1 + r)] + \\ &\quad [A / (1 + r)^2] + [A / (1 + r)^3] \\ &\quad + \dots \\ &= A / r\end{aligned}$$

Suppose  $A = 100$ ;  $r = 0.1$

$$PV = 100 / 0.1 = 1000$$

## ❖ Annuity for T years

An amount A is received annually one year from now for T years.

$$PV = [ A / (1 + r) ] + [ A / (1 + r)^2 ] \\ + \dots + [ A / (1 + r)^T ]$$

$$= A [ (1 + r)^T - 1 ] / [ (1 + r)^T (r) ]$$

Suppose  $A = 100$ ;  $r = 0.1$ ;  $T = 8$

$$PV = 100 [ (1.1)^8 - 1 ] / [ (1.1)^8 (0.1) ] \\ = 533.49$$

## ❖ Annuity Growing into Perpetuity

An amount  $A$  is received one year from now.

From the end of second year onwards the amount received at the end of a year is  $(1 + g)$  times the amount received at the end of the previous year.



Assume the growth factor  $g$  is less than the discount rate  $r$ .

$$PV = [A / (1 + r)] + [A (1+g) / (1+r)^2] + [A (1+g)^2 / (1+r)^3] + \dots = A / (r-g)$$

Suppose  $A = 100$ ;  $r = 0.1$  and  $g = 0.05$

$$PV = 100 / (0.1 - 0.05) = 100 / 0.05 = 2000$$

# Multi-period investment

❖ Investment of Rs.1000

Amount received after one year  
= Rs.600

Amount received after two years  
= Rs.800

Suppose the discount rate is  
10%.

Present value (PV) of future cash flows

$$\begin{aligned} &= (600) / (1.10) + (800) / (1.10)^2 \\ &= 1206.6 \end{aligned}$$

Net Present Value = PV of future cash flows – Investment cost

$$\begin{aligned} &= 1206.6 - 1000 \\ &= 206.6 \end{aligned}$$

# Capital Budgeting

- ❖ Projects classified into specific areas / buckets
- ❖ Projects in different buckets may be evaluated using different criteria
- ❖ Within each bucket rank order projects using the same criteria
- ❖ Buckets may be further sub-divided

	Rs. Crores		Rs. Crores
Core Technology Development	10.00	New Product Development	12.00
Project 4	4.60	Project 3	4.50
Project 2	5.40	Project 1	2.70
		Project 6	4.80

## ❖ Selection Criteria

- Financial

- Non-Financial

## ❖ Financial Models

- ✓ Single criteria

- ✓ Easy to use

## ❖ Payback period

- Requires estimation of cash flows

➤ Example

Initial Cost = 100,000

Annual savings or incremental net  
cash in flow = 25,000

Payback =  $100,000 / 25,000 = 4$

## Another Example:

Year	1	2	3
Incremental Net Cash Inflow Rs.	30,000	24,000	20,000

$$\begin{aligned}\text{Pay back period} &= 1 + 20,000 / 24,000 \\ &= 1 \frac{5}{6} \text{ years}\end{aligned}$$



# Discounted Payback:

Discount Rate = 10%

Initial investment = 50,000

Discounted Cash flows :

Year 1:  $30,000 / 1.1 = 27,272.13$  ;

Year 2:  $24,000 / (1.1)^2 = 19,834.71$  ;

Year 3:  $20,000 / (1.1)^3 = 15,026.30$

Discounted Payback

$$= 2 + [ 50,000 - (27,272.73 + 19,834.71) ] / 15,026.3$$

$$= 2.1925$$

- ❖ Accept the project if payback / discounted payback period is less than a specified value
- ❖ Incremental Net Cash flows after payback/ discounted payback period are ignored
- ❖ May be used as a screening device if long payback/discounted payback is not acceptable
- ❖ Estimation of incremental net cash flows over a long time horizon may be highly uncertain / difficult

## ❖ Net Present value

- Requires estimation of cash flows and cost of capital

$I_0$  : Initial investment (negative because it is an outflow)

$F_t$  : Incremental Net Cash flow for period  $t$  (negative if net cash flow is an outflow)

$k$  : required rate of return

$n$  : Life of the output of the project

- Note:  $F_n$  will include the salvage value of the assets created by the project
- Project NPV

$$= I_0 + \sum_{t=1}^n F_t / (1 + k)^t$$

$k$  is typically the weighted average cost of capital (WACC)

Alternatively  $k$  may depend on the riskiness of the project

- Previous Example
- Suppose  $k = 10\%$
- $$\begin{aligned} \text{NPV} &= -50,000 + 30,000 / 1.1 \\ &\quad + 24,000 / (1.1)^2 + 20,000 / (1.1)^3 \\ &= -50,000 + 27,273 + 19,835 \\ &\quad + 15,206 \\ &= 12,134 \end{aligned}$$
- If  $\text{NPV} > 0$ , accept the project
- Ignores size of initial investment

## ➤ Internal Rate of Return

$I_0$  : Initial investment (negative because it is an outflow)

$F_t$  : Incremental Net Cash flow for period  $t$  (negative if net cash flow is an outflow)

$k$  : Internal rate of return

$n$  : Life of the output of the project

$F_n$  : includes the salvage value of the assets created by the project

➤ Find  $k$  such that

$$I_0 + \sum_{t=1}^n F_t / (1+k)^t = 0$$

$k$  is found by trial and error

- ❖ Accept project if  $k > \text{hurdle rate}$
- ❖ Requires estimation of cash flows and hurdle rate
- ❖ Possibility of multiple internal rates of return if some of the future cash flows,  $F_t$ , are negative i.e. are outflows



❖ Previous Example

❖ Find  $k$  so that

$$\begin{aligned} & - 50,000 + 30,000 / (1+k) \\ & + 24,000 / (1+k)^2 + 20,000 / (1+k)^3 \\ & = 0 \end{aligned}$$

By trial and error, IRR is approximately  
24.42%

## ❖ **Benefit Cost Ratio**

- ✓ **Also called Profitability Index**
- ✓ **= (present value of all future incremental cash flows) / (Initial Investment)**
- ✓ **Accept project if this ratio is > 1**

## ✓ Previous Example

Present Value of Incremental cash flows

$$= 30,000 / 1.1 + 24,000 / (1.1)^2 + 20,000 / (1.1)^3 = 62,134$$

$$\begin{aligned} \text{✓ Benefit Cost Ratio} &= 62,134 / 50,000 \\ &= 1.2427 \end{aligned}$$

- ❖ Mutually exclusive projects
- ❖ Only one of the projects A or B can be undertaken
  - Different ways of extending the capacity of warehouse

For payback / discounted payback criterion accept the project with least payback / discounted payback period if it is less than some specified value

For NPV, accept the project with highest NPV if it is greater than 0

- For IRR, accept the project with the highest IRR if it is greater than the hurdle rate
- For Benefit Cost Ratio, accept the project with the highest Benefit Cost Ratio if it is greater than 1

- Contingent projects

- ✓ Project B can be accepted only if Project A is accepted

- ✓ Now we have two mutually exclusive cases

Project A only or Project A + Project B

- ❖ Resource constrained projects

- NPV criterion

- Optimization model

- ✓ Integer programming

# Inflation

- Suppose nominal interest rate = 10% per annum.

Suppose inflation is estimated to be 7% per annum.

The real interest rate is approximately  
= Nominal interest rate – inflation rate  
= 10 – 7 = 3%

The exact real rate is calculated as follows:

Rs.1 invested will at the end of the year grow to  $(1 + \text{nominal interest rate})$

But purchasing power of this amount is only  $[(1 + \text{nominal rate})] / (1 + \text{inflation rate})$



So the real gain or real rate of interest  
=  $[(1 + \text{nominal rate}) / (1 + \text{inflation rate})] - 1$

The exact real interest rate

$$= (1 + 0.1) / (1 + 0.07) - 1$$

$$= (1.1) / (1.07) - 1 = 0.02804 \text{ or } 2.804\%$$

Note: The approximation is good only if the nominal rate and inflation rate are small.

Suppose nominal interest rate = 100%  
and inflation rate = 80%

Approximate real interest rate  
 $= 100 - 80 = 20\%$

The exact real interest rate  
 $= (1 + 1) / (1 + 0.8) - 1$   
 $= (2 / 1.8) - 1$   
 $= 0.1111 \quad \text{or} \quad 11.11\%$

Note: Nominal cash flows should be discounted at nominal rates.

Real cash flows should be discounted at real rates.

Previous example:

Initial cost = -50,000

Nominal discount rate = 10%

Estimated inflation rate = 7%

Real rate =  $(1 + 0.1) / (1 + 0.07) - 1$   
= 0.02804 or 2.804%

Year	1	2	3
Incremental Net Cash Inflow (Nominal)	30,000	24,000	20,000

Incremental Net Cash Inflow (Real):

$$\text{Year 1: } 30,000 / 1.07 = 28,037.38$$

$$\text{Year 2: } 24,000 / (1.07)^2 = 20,962.53$$

$$\text{Year 3: } 20,000 / (1.07)^3 = 16,325.96$$

Discounting at real rate of 2.804%

Present Value of Cash flows:

$$\text{Year 1: } 28,037.38 / 1.02804 = 27,272.66$$

$$\begin{aligned} \text{Year 2: } 20,962.53 / (1.02804)^2 \\ = 19,834.61 \end{aligned}$$

$$\begin{aligned} \text{Year 3: } 16,325.96 / (1.02804)^3 \\ = 15,026.18 \end{aligned}$$

Net Present Value

$$\begin{aligned} = & - 50,000 + 27,272.66 + 19,834.61 \\ & + 15,026.18 = 12,133.45 \approx 12,134 \end{aligned}$$

# Cost Concepts

- Depreciation
  - Initial investment of Rs.1,00,000 is depreciated over 5 years say using straight line depreciation.

For accounting purpose,  
 $(1,00,000) / 5 = 20,000$  is treated as depreciation.

For capital budgeting, the entire amount of Rs.1,00,000 is initial investment cost.

- Sunk Costs

- You have hired a consultant and paid Rs.10,00,000 last year to identify strategic growth plan for your organisation.

Now you want to evaluate projects that have been identified.

The amount paid to consultant is a sunk cost.

- Opportunity Cost
  - You have an empty warehouse which could be sold. Now you are considering a project which can utilise the warehouse space.

The selling price of the warehouse is an opportunity cost for the project.



## ❖ Disadvantages of Financial Models

- Ignores all non-monetary factors
- Biased towards short run
- Difficult to forecast incremental cash flows
- Sensitive to errors in estimating incremental cash flows during the early years

- Difficult to estimate the life of the output of the project and the salvage value of the assets created by the project
- The effect of changes (or errors) in incremental cash flows and / or cost of capital is not easy to comprehend

# Multi-criteria Models

- ❖ Uses several criteria
  - ❖ Vary in complexity
  - ❖ Different information requirements
  - ❖ Relevant factors or criteria are selected
  - ❖ Unweighted 0-1 factor model
    - ❖ For each factor, the rater / decision maker scores the project in terms of qualifies or not qualifies

- ❖ The number of qualifying factors is calculated

  - If this number is sufficient, the project is selected

- ❖ Disadvantage is that all factors are given equal importance

- ❖ No gradation of the degree to which the factor is satisfied

Project _____	Date _____		
Rater _____		Qualifies	Does Not Qualify
No increase in energy requirements		x	
Potential market size, dollars		x	
Potential market share, percent		x	
No new facility required			x
No new technical expertise required		x	
No decrease in quality of final product			x
Ability to manage project with current personnel		x	
No requirement for reorganization		x	
Impact on work force safety		x	
Impact on environmental standards			
Profitability		x	
Rate of return more than 15% after tax		x	
Estimated annual profits more than \$250,000		x	
Time to break-even less than 3 years			x
Need for external consultants			x
Consistency with current line of business			
Impact on company image		x	
With customers			x
With our industry			
Totals		12	5

Figure 2-2 Sample project evaluation form.

Figure 2.2 from Project Management: A Managerial Approach, Seventh Edition, John Wiley & Sons 2009, Jach R. Meredith and Samuel J Mantel

## ❖ Unweighted Factor Scoring Model

- A numerical score is given to the degree to which each criterion is met
- Typically a five point scale is used
- For each criteria, five ranges of performance level are identified
  - ✓ Typically the ranges are chosen so that from past experience, roughly equal number of projects will fall in each range

✓ It is not necessary to have equal ranges

✓ Example

Score	NPV
5	Above 1,000,000
4	750,000 to 1,000,000
3	400,00 to 750,00
2	100,000 to 400,000
1	Less than 100,000

Score	Ability to manage project with current personnel
-------	---

- |   |                                   |
|---|-----------------------------------|
| 5 | Very easy and definitely possible |
| 4 | Easy without much effort          |
| 3 | Possible with extra effort        |
| 2 | Difficult                         |
| 1 | Very difficult                    |



- ❖ For each project, the scores for the different criteria are added to get the total score
- ❖ The projects are ranked based on the total score
- ❖ Assumes all criteria are equally important

## ❖ Weighted Scoring Model

$s_{ij}$  : Score of the  $i^{\text{th}}$  project on  $j^{\text{th}}$  criteria

$w_j$  : Weight for  $j^{\text{th}}$  criterion

$n$  : Number of factors or criteria

$R_i$  : Total score for  $i^{\text{th}}$  project

$$R_i = \sum_{j=1}^n w_j s_{ij}$$

- Desirable to normalize the weights so that

$$\sum_{j=1}^n w_j = 1$$

- This would give an idea about the relative importance of the criteria
- How to calculate the weights / relative importance of criteria
  - ✓ Internal team
  - ✓ Analytic Hierarchy Process

Thank You